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On the strong coupling region in quantum matrix string theory

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Abstract: We study the behavior of matrix string theory in the strong coupling region, where matrix strings reduce to discrete light-cone type-IIA superstrings except at the usual string-interaction points. In the large-N limit, this reduction corresponds to the double-dimensional reduction from wrapped supermembranes on $R^{10} \times S^1$ to type-IIA superstrings on R^{10} in the light-cone gauge. Such reductions were shown classically, while they are not obvious quantum mechanically. Recently, Sekino and Yoneya analyzed the double-dimensional reduction of the wrapped supermembrane quantum mechanically to one-loop order in the strong coupling expansion. We analyze the problem in matrix string theory by using the same expansion. At the one-loop level, the quantum corrections cancel out as was presented by them. However, at the two-loop level we find that the quantum corrections cancel out only for the leading terms in the large N.

Keywords: Strong Coupling Expansion, M-Theory, String Duality, M(atrix) Theories.

Contents

1. Introduction

It is believed that the supermembrane in eleven dimensions [1, 2] plays an important role to understand the fundamental degrees of freedom in M-theory which is a unified description of the various superstring theories. Actually, it was shown that the supermembrane in eleven dimensions is related to type-IIA superstring in ten dimensions by means of the classical double-dimensional reduction [3]. The procedure is the following: (i) Consider the target space of $R^{10} \times S^1$. (ii) Set the compactified coordinate (with radius L) proportional to one of the spatial coordinates of the world volume, which we call ρ coordinate. (iii) Simply ignore the infinite tower of the Kaluza-Klein (non-zero) modes. However, it is not obvious whether such a reduction is justified also in quantum theory. Actually, it was pointed out that the other set of zero-mode states which are independent of the other spatial coordinate of the world volume, which we call σ coordinate, do not decouple even in the zero-radius limit ($L \to 0$) [4]. Hence, whether the Kaluza-Klein modes along the compactified ρ direction are suppressed quantum mechanically seems to be a subtle question.

Sekino and Yoneya analyzed the double-dimensional reduction quantum mechanically with the light-cone supermembrane action in the appendix of their paper [5]. Contrary to the classical treatments, they kept the Kaluza-Klein modes associated with the ρ coordinate in the wrapped supermembrane theory on the target space $R^{10} \times S^1$ and they integrated them out by using the perturbative expansion with respect to the radius L. Since the gauge coupling satisfies $g \sim 1/L$ in the wrapped supermembrane theory, the expansion can be regarded as the strong coupling expansion. They calculated the effective action for the zero modes along the ρ direction to the one-loop order of $O(L^2)$ by integrating out the Kaluza-Klein modes. They found that the quantum corrections cancel out and the effective action agrees with the classical (free) action of type-IIA superstring except at the points where the usual string interactions could occur. However, as is emphasized in their paper [5], the strong coupling expansion does not give a rigorous proof of the quantum double-dimensional reduction. The free parts of the Kaluza-Klein modes in the strong coupling expansion have no derivatives and it leads to the propagators which are proportional to the two-dimensional δ-function, $\delta^{(2)}(\xi) \equiv \delta(\tau)\delta(\sigma)$. Thus, the loop diagrams suffer from the ultraviolet divergences of $\delta^{(2)}(0)$ type, and we need a regularization for a rigorous treatment. However it is very difficult to find a suitable regularization which respects symmetries (e.g., supersymmetry and gauge symmetry), and hence the strong coupling expansion is not defined rigorously. In this sense, they gave a formal argument for the vanishing of the one-loop corrections of $O(L^2)$ by demonstrating that the coefficients of $\delta^{(2)}(0)$ coming from both bosonic and fermionic degrees of freedom cancel out.

The purpose of this paper is essentially to extend their (formal) calculations to the two-loop order of $O(L^2)$. However, the naive extension is not straightforward because at the two-loop level, even the coefficients of the $\delta^{(2)}(0)$ diverge due to the contribution of the infinite Kaluza-Klein towers. Thus, we need another regularization for the summation over the infinite tower of the Kaluza-Klein modes at the two-loop level. Contrary to the case of the divergence of $\delta^{(2)}(0)$ itself, it is relatively easy to find a regularization (which respects symmetries) for the divergence of the coefficients due to the infinite Kaluza-Klein tower along the compactified ρ direction. In fact, we know the matrix regularization of the supermembrane on R^{11} in the light-cone gauge [6] and also that of the wrapped supermembrane on $R^{10} \times S^1$ in the light-cone gauge [5].[1] The former is called Matrix theory [7] which was proposed to be a non-perturbative formulation of light-cone quantized M-theory in the large-N limit and the latter is called matrix string theory [8, 9] which will be a non-perturbative formulation of light-cone quantized type-IIA superstring theory in the large-N limit. Furthermore, even at finite N, Matrix and matrix string theories are conjectured to be non-perturbative formulations of discrete light-cone quantized (DLCQ)[2] M-theory and type-IIA superstring theory, respectively [10, 11, 12]. Thus, in this paper we consider matrix string theory and study whether the reduction from matrix strings to discrete light-cone type-IIA superstrings is justified quantum mechanically. According to the correspondence of the wrapped supermembrane with matrix string [5], the zero modes along the ρ direction, i.e., type-IIA superstring degrees of freedom in the wrapped supermembrane theory, are mapped to the diagonal elements in matrix string theory. Hence, in

[1]In ref. [5], the quantum mechanical study on the double-dimensional reduction is discussed in appendix A, and in the body of the paper, the correspondence of the degrees of freedom in the wrapped supermembrane theory with those in matrix string theory is discussed in detail.

[2]In this paper we use a convention of the light-cone coordinates such that $x^{\pm} = (x^0 \pm x^{10})/\sqrt{2}$. Furthermore, x^- is compactified on S^1 with radius R in DLCQ.

this paper we study whether the reduction from matrix strings to the diagonal elements of strings is justified quantum mechanically to the two-loop order of $O(L^2)$, except at the points where the strings could interact usually.

The plan of this paper is as follows. In the next section, we review the correspondence of the wrapped supermembranes on $R^{10} \times S^1$ with matrix strings. In section 3 we discuss the strong coupling expansion in matrix string theory. By using the expansion in the path-integral formula, we integrate out the off-diagonal matrix elements to the two-loop order of $O(L^2)$. We obtain the effective action for the diagonal matrix elements and study whether the reduction from matrix strings to the diagonal elements is quantum mechanically justified or not. Section 4 is devoted to our conclusion and discussion. In appendix A, we put the explicit expressions of the interaction parts of the action.

2. From wrapped supermembrane to matrix string

Our starting point is the following light-cone gauge fixed supermembrane action on the target space R^{11} [6],

$$S = LT \int d\tau \int_0^{2\pi} d\sigma d\rho \left[\frac{1}{2}(D_\tau X^i)^2 - \frac{1}{4L^2}\{X^i, X^j\}^2 + i\psi^T D_\tau \psi + \frac{i}{L}\psi^T \gamma^i \{X^i, \psi\} \right], \quad (2.1)$$

$$\begin{aligned} D_\tau X^i &= \partial_\tau X^i - \frac{1}{L}\{A, X^i\}, \\ D_\tau \psi &= \partial_\tau \psi - \frac{1}{L}\{A, \psi\}, \\ \{A, B\} &= \partial_\sigma A\, \partial_\rho B - \partial_\rho A\, \partial_\sigma B, \end{aligned}$$

where the indices i, j run through $1, 2, \ldots, 9$, the spinor ψ has sixteen real components[3] and T is the membrane tension. At this stage, L is an arbitrary length parameter of no physical meaning. This action is invariant under the gauge transformation,

$$\delta A = \partial_\tau \Lambda + \frac{1}{L}\{\Lambda, A\}, \quad (2.2)$$

$$\delta X^i = \frac{1}{L}\{\Lambda, X^i\}, \quad (2.3)$$

$$\delta \psi = \frac{1}{L}\{\Lambda, \psi\}. \quad (2.4)$$

This gauge transformation generates the area-preserving diffeomorphism on the world volume. In the $A = 0$ gauge, the Gauss law constraint derived from the action (2.1) is given by

$$\{\partial_\tau X^i, X^i\} + i\{\psi^T, \psi\} = 0. \quad (2.5)$$

This constraint is originally the integrability condition for the equations determining the light-cone coordinate X^-,

$$\frac{1}{LT} P^+ \partial_{\hat\sigma} X^- = \partial_{\hat\sigma} X^i \partial_\tau X^i + i\psi^T \partial_{\hat\sigma}\psi, \quad (2.6)$$

[3] We use the real and symmetric representation for the gamma matrices γ^i, which satisfy $\{\gamma^i, \gamma^j\} = 2\delta^{ij}$.

where $\hat{\sigma} = (\sigma, \rho)$ and this equation is locally equivalent to eq. (2.5). Note that the light-cone momentum (density), $P^+ = LT\partial_\tau X^+$, is constant on the world volume. When the spatial surface of the supermembrane has a non-trivial topology, we have to impose further the global constraints. Actually, in the case of the toroidal supermembrane,[4] the global constraints are given by

$$\int_0^{2\pi} d\sigma(\partial_\sigma X^i \partial_\tau X^i + i\psi^T \partial_\sigma \psi) = \int_0^{2\pi} d\rho(\partial_\rho X^i \partial_\tau X^i + i\psi^T \partial_\rho \psi) = 0\,. \tag{2.7}$$

Now, we consider the wrapped supermembrane theory on the target space $R^{10} \times S^1$ and discuss the correspondence of the wrapped supermembrane with matrix string [5]. We take the X^9 direction as the S^1 and identify the radius with the above parameter L,

$$X^9 = L\rho + Y\,. \tag{2.8}$$

Thus L has the physical meaning of a radius of the X^9 direction which is regarded as the "eleventh" direction in M-theory. Substituting eq. (2.8) into eq. (2.1), we obtain the following light-cone gauge fixed supermembrane action on $R^{10} \times S^1$,

$$\begin{aligned} S = LT \int d\tau \int_0^{2\pi} d\sigma d\rho \Big[&\frac{1}{2}F_{\tau\sigma}^2 + \frac{1}{2}(D_\tau X^k)^2 - \frac{1}{2}(D_\sigma X^k)^2 - \frac{1}{4L^2}\{X^k, X^l\}^2 + \\ &+ i\psi^T D_\tau \psi - i\psi^T \gamma^9 D_\sigma \psi + \frac{i}{L}\psi^T \gamma^k \{X^k, \psi\} \Big]\,, \end{aligned} \tag{2.9}$$

$$\begin{aligned} F_{\tau\sigma} &= \partial_\tau Y - \partial_\sigma A - \frac{1}{L}\{A, Y\}\,, \\ D_\sigma X^k &= \partial_\sigma X^k - \frac{1}{L}\{Y, X^k\}\,, \\ D_\sigma \psi &= \partial_\sigma \psi - \frac{1}{L}\{Y, \psi\}\,, \end{aligned}$$

where the indices k, l run through $1, 2, \ldots, 8$. This is also an action of the gauge theory of the area-preserving diffeomorphism, where the gauge coupling $g \sim 1/L$. The gauge transformations are as follows,

$$\delta A = \partial_\tau \Lambda + \frac{1}{L}\{\Lambda, A\}\,, \tag{2.10}$$

$$\delta Y = \partial_\sigma \Lambda + \frac{1}{L}\{\Lambda, Y\}\,, \tag{2.11}$$

$$\delta X^k = \frac{1}{L}\{\Lambda, X^k\}\,, \tag{2.12}$$

$$\delta \psi = \frac{1}{L}\{\Lambda, \psi\}\,. \tag{2.13}$$

Furthermore, substituting eq. (2.8) into eqs. (2.7), we have the global constraints

$$\int_0^{2\pi} d\sigma(\partial_\sigma Y \partial_\tau Y + \partial_\sigma X^k \partial_\tau X^k + i\psi^T \partial_\sigma \psi) = 0\,, \tag{2.14}$$

$$\int_0^{2\pi} d\rho(L\partial_\tau Y + \partial_\rho Y \partial_\tau Y + \partial_\rho X^k \partial_\tau X^k + i\psi^T \partial_\rho \psi) = 0\,. \tag{2.15}$$

[4]In this paper we consider this case only.

In ref. [5], the infinite dimensional gauge group of the area-preserving diffeomorphism in eq. (2.9) was regularized by the finite dimensional group U(N) and it was shown that the matrix-regularized form of the action (2.9) agrees with that of matrix string theory,

$$
\begin{aligned}
S = LT \int d\tau \int_0^{2\pi} d\theta\, \mathrm{tr} \Big[& \frac{1}{2} F_{\tau\theta}^2 + \frac{1}{2}(D_\tau X^k)^2 - \frac{1}{2}(D_\theta X^k)^2 + \frac{1}{4L^2}[X^k, X^l]^2 + \\
& + i\psi^T D_\tau \psi - i\psi^T \gamma^9 D_\theta \psi - \frac{1}{L}\psi^T \gamma^k [X^k, \psi] \Big] ,
\end{aligned} \tag{2.16}
$$

$$
\begin{aligned}
F_{\tau\theta} &= \partial_\tau Y - \partial_\theta A - \frac{i}{L}[A, Y] , \\
D_\tau X^k &= \partial_\tau X^k - \frac{i}{L}[A, X^k] , \\
D_\theta X^k &= \partial_\theta X^k - \frac{i}{L}[Y, X^k] ,
\end{aligned}
$$

where each element of the matrices is a function of τ and θ. Note that the action (2.16) can be derived from Matrix theory action by combining T- and S-dualities with the flipping of the compactified direction from eleventh to ninth [8, 9]. The U(N) gauge transformations of the action (2.16) are as follows,

$$
\delta A = \partial_\tau \Lambda + \frac{i}{L}[\Lambda, A] , \tag{2.17}
$$

$$
\delta Y = \partial_\theta \Lambda + \frac{i}{L}[\Lambda, Y] , \tag{2.18}
$$

$$
\delta X^k = \frac{i}{L}[\Lambda, X^k] , \tag{2.19}
$$

$$
\delta \psi = \frac{i}{L}[\Lambda, \psi] . \tag{2.20}
$$

In the correspondence between the actions (2.9) and (2.16), the zero-modes along the ρ direction in the wrapped supermembrane are mapped to the diagonal elements of matrix string and the Kaluza-Klein modes are mapped to the off-diagonal elements [5]. Here, we should notice that in the matrix regularization of the wrapped supermembrane on $R^{10} \times S^1$, we have no obvious counterparts of the global constraints (2.14) and (2.15), because the (matrix-regularized) Gauss law constraint, which is derived from eq. (2.16), cannot be manifestly interpreted as the integrability condition.[5] Furthermore, in the standard derivation [8, 9] of matrix string theory based on Seiberg and Sen's arguments [11, 12] and the compactification prescription of Taylor [13], such global constraints do not appear naturally.[6]

The classical double-dimensional reduction is to assume that the Kaluza-Klein modes along the ρ direction of every field are zero. Then the action (2.9) reduces to

$$
S = 2\pi LT \int d\tau \int_0^{2\pi} d\sigma \left[\frac{1}{2}(\partial_\tau X^k)^2 - \frac{1}{2}(\partial_\sigma X^k)^2 + i\psi^T \partial_\tau \psi - i\psi^T \gamma^9 \partial_\sigma \psi \right] , \tag{2.21}
$$

[5] Of course, also in the matrix regularization of the supermembrane on R^{11} [6], i.e., Matrix theory, we have no obvious counterparts of the global constraints (2.7) due to the same reason.

[6] In ref. [8], the origin of the level-matching condition in matrix string theory was discussed.

where, for simplicity, we also set the zero modes of A and Y fields to zero. With the identification $2\pi LT = 1/2\pi\alpha'$, which is kept finite in the $L \to 0$ limit, this action agrees with the light-cone type-IIA superstring action in the Green-Schwarz formalism. In the matrix-regularized action (2.16), such a classical double-dimensional reduction corresponds to the assumption that the off-diagonal elements of every matrix are zero,

$$X^k = \begin{pmatrix} x_1^k & & & 0 \\ & x_2^k & & \\ & & \ddots & \\ 0 & & & x_N^k \end{pmatrix}, \qquad \psi = \begin{pmatrix} \psi_1 & & & 0 \\ & \psi_2 & & \\ & & \ddots & \\ 0 & & & \psi_N \end{pmatrix}. \tag{2.22}$$

Then the action reduces to the DLCQ type-IIA superstring action in the light-cone momentum $p^+ = N/R$ sector. Depending on the boundary conditions with respect to θ, the diagonal elements $x_a^k(\theta)$ $(a = 1, \dots, N)$ in the matrix (2.22) describe one or more separate strings. For example, the boundary conditions $x_a^k(\theta + 2\pi) = x_a^k(\theta)$ correspond to N string bits having $p^+ = 1/R$, which are regarded as the minimal length strings in DLCQ. On the other hand, a string of maximal length having $p^+ = N/R$ is described by the boundary condition $x_a^k(\theta + 2\pi) = x_{a+1}^k(\theta)$, $x_{N+1}^k(\theta) = x_1^k(\theta)$.

It is expected that the above reductions are justified also in quantum theory.[7] However, as was discussed in refs. [4] and [5],[8] the justification is not so simple. In particular, in the appendix of ref. [5], the quantum double-dimensional reduction of the wrapped supermembrane (2.9) was analyzed for the small radius L, which corresponds to the strong gauge coupling $g \sim 1/L$ in the wrapped supermembrane theory and also to the weak string coupling $g_s \sim L/\sqrt{\alpha'}$ in type-IIA superstring theory. Concretely, by using the perturbative expansion with respect to L in the path-integral formula, the Kaluza-Klein modes along the ρ direction were integrated out to the one-loop order of $O(L^2)$, and it was found that the effective action for the zero modes agrees with the classical (free) action of the type-IIA superstring except at the points where perturbative interactions would occur by joining or splitting of strings. That result is consistent with the expectation that the wrapped supermembrane theory in the region of small radius L agrees with the perturbative type-IIA superstring theory. In the next section, we analyze the quantum reduction of matrix string (2.16) to the diagonal elements for small radius L. That is, by using the same perturbative expansion in the path-integral formula, we integrate out the off-diagonal matrix elements to the two-loop order of $O(L^2)$ and study whether the effective action for the diagonal matrix elements agrees with the classical (free) action of the DLCQ type-IIA superstring except at the points where perturbative interactions would occur by joining or splitting of DLCQ strings.

[7]To be precise, since it is not obvious whether the quantum supermembranes can be the degrees of freedom in M-theory, there may be no logical reasons for the expectation that the reduction from the wrapped supermembranes to type-IIA superstrings is justified also in quantum theory. However, in the context of Matrix and matrix string theories, the quantum matrix-regularized supermembranes are the degrees of freedom in DLCQ M-theory. Hence, it is expected that the reduction from matrix strings to DLCQ type-IIA strings is justified also in quantum theory.

[8]See also ref. [14] for related discussions.

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3. Strong coupling expansion in matrix string theory

3.1 Path-integral formula

To begin with, we decompose every $N \times N$ hermite matrix in eq. (2.16) into the diagonal and off-diagonal parts as follows,

$$A \to a + A\,, \tag{3.1}$$
$$Y \to y + Y\,, \tag{3.2}$$
$$X^k \to x^k + X^k\,, \tag{3.3}$$
$$\psi \to \psi + \Psi\,, \tag{3.4}$$

where a, y, x^k and ψ are the diagonal parts;

$$a = (a_a) = \begin{pmatrix} a_1 & & & 0 \\ & a_2 & & \\ & & \ddots & \\ 0 & & & a_N \end{pmatrix}, \qquad y = (y_a) = \begin{pmatrix} y_1 & & & 0 \\ & y_2 & & \\ & & \ddots & \\ 0 & & & y_N \end{pmatrix},$$
$$x^k = (x^k_a) = \begin{pmatrix} x^k_1 & & & 0 \\ & x^k_2 & & \\ & & \ddots & \\ 0 & & & x^k_N \end{pmatrix}, \qquad \psi = (\psi_a) = \begin{pmatrix} \psi_1 & & & 0 \\ & \psi_2 & & \\ & & \ddots & \\ 0 & & & \psi_N \end{pmatrix}, \tag{3.5}$$

and A, Y, X^k and Ψ are the off-diagonal parts,

$$A = \Big(A_{ab}\Big) = \begin{pmatrix} 0 & A_{12} & \cdots & A_{1N} \\ A_{21} & 0 & \cdots & A_{2N} \\ \vdots & & \ddots & \\ A_{N1} & A_{N2} & \cdots & 0 \end{pmatrix}, \quad Y = \Big(Y_{ab}\Big) = \begin{pmatrix} 0 & Y_{12} & \cdots & Y_{1N} \\ Y_{21} & 0 & \cdots & Y_{2N} \\ \vdots & & \ddots & \\ Y_{N1} & Y_{N2} & \cdots & 0 \end{pmatrix},$$
$$X^k = \Big(X^k_{ab}\Big) = \begin{pmatrix} 0 & X^k_{12} & \cdots & X^k_{1N} \\ X^k_{21} & 0 & \cdots & X^k_{2N} \\ \vdots & & \ddots & \\ X^k_{N1} & X^k_{N2} & \cdots & 0 \end{pmatrix},$$
$$\Psi = \Big(\Psi_{ab}\Big) = \begin{pmatrix} 0 & \Psi_{12} & \cdots & \Psi_{1N} \\ \Psi_{21} & 0 & \cdots & \Psi_{2N} \\ \vdots & & \ddots & \\ \Psi_{N1} & \Psi_{N2} & \cdots & 0 \end{pmatrix}. \tag{3.6}$$

After the replacement of eqs. (3.1)–(3.4), the action (2.16) becomes

$$S = LT \int d\tau \int_0^{2\pi} d\theta (\mathcal{L}_B + \mathcal{L}_F)\,, \tag{3.7}$$
$$\mathcal{L}_B = \mathrm{tr}\left[\frac{1}{2}(\partial_\theta a)^2 - \partial_\theta a \partial_\tau y + \frac{1}{2}(\partial_\tau y)^2 + \frac{1}{2}(\partial_\tau x^k)^2 - \frac{1}{2}(\partial_\theta x^k)^2 - \right.$$
$$- \frac{1}{2L^2}([a, Y] + [A, y])^2 - \frac{1}{2L^2}([a, X^k] + [A, x^k])^2 +$$

$$
\begin{aligned}
&+\frac{1}{2L^2}([y,X^k]+[Y,x^k])^2+\frac{1}{4L^2}([x^k,X^l]+[X^k,x^l])^2-\\
&-\frac{i}{L}(\partial_\tau Y-\partial_\theta A)([a,Y]+[A,y])-\frac{i}{L}(\partial_\tau y-\partial_\theta a)[A,Y]-\\
&-\frac{i}{L}\partial_\tau X^k([a,X^k]+[A,x^k])-\frac{i}{L}\partial_\tau x^k[A,X^k]+\\
&+\frac{i}{L}\partial_\theta X^k([y,X^k]+[Y,x^k])+\frac{i}{L}\partial_\theta x^k[Y,X^k]-\\
&-\frac{1}{L^2}([a,Y]+[A,y])[A,Y]-\frac{1}{L^2}([a,X^k]+[A,x^k])[A,X^k]+\\
&+\frac{1}{L^2}([y,X^k]+[Y,x^k])[Y,X^k]+\frac{1}{2L^2}([x^k,X^l]+[X^k,x^l])[X^k,X^l]+\\
&+\frac{1}{2}(\partial_\tau Y-\partial_\theta A)^2+\frac{1}{2}(\partial_\tau X^k)^2-\frac{1}{2}(\partial_\theta X^k)^2-\\
&-\frac{i}{L}\partial_\tau Y[A,Y]+\frac{i}{L}\partial_\theta A[A,Y]-\frac{i}{L}\partial_\tau X^k[A,X^k]+\frac{i}{L}\partial_\theta X^k[Y,X^k]+\\
&\left.-\frac{1}{2L^2}[A,Y]^2-\frac{1}{2L^2}[A,X^k]^2+\frac{1}{2L^2}[Y,X^k]^2+\frac{1}{4L^2}[X^k,X^l]^2\right], \qquad (3.8)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_F=\operatorname{tr}\Big[&i\psi^T\partial_\tau\psi-i\psi^T\gamma^9\partial_\theta\psi+\frac{1}{L}\Psi^T[a,\Psi]-\frac{1}{L}\Psi^T\gamma^9[y,\Psi]-\frac{1}{L}\Psi^T\gamma^k[x^k,\Psi]+\\
&+\frac{2}{L}\psi^T[A,\Psi]-\frac{2}{L}\psi^T\gamma^9[Y,\Psi]-\frac{2}{L}\psi^T\gamma^k[X^k,\Psi]+i\Psi^T\partial_\tau\Psi-\\
&-i\Psi^T\gamma^9\partial_\theta\Psi+\frac{1}{L}\Psi^T[A,\Psi]-\frac{1}{L}\Psi^T\gamma^9[Y,\Psi]-\frac{1}{L}\Psi^T\gamma^k[X^k,\Psi]\Big]. \qquad (3.9)
\end{aligned}
$$

Furthermore, the gauge transformations are decomposed as

$$\delta a=\partial_\tau\lambda+\frac{i}{L}[\Lambda,A]_{\text{diag}}\,, \qquad (3.10)$$

$$\delta A=\partial_\tau\Lambda+\frac{i}{L}([\lambda,A]+[\Lambda,a]+[\Lambda,A]_{\text{off-diag}})\,, \qquad (3.11)$$

$$\delta y=\partial_\theta\lambda+\frac{i}{L}[\Lambda,Y]_{\text{diag}}\,, \qquad (3.12)$$

$$\delta Y=\partial_\theta\Lambda+\frac{i}{L}([\lambda,Y]+[\Lambda,y]+[\Lambda,Y]_{\text{off-diag}})\,, \qquad (3.13)$$

$$\delta x^k=\frac{i}{L}[\Lambda,X^k]_{\text{diag}}\,, \qquad (3.14)$$

$$\delta X^k=\frac{i}{L}([\lambda,X^k]+[\Lambda,x^k]+[\Lambda,X^k]_{\text{off-diag}})\,, \qquad (3.15)$$

$$\delta\psi=\frac{i}{L}[\Lambda,\Psi]_{\text{diag}}\,, \qquad (3.16)$$

$$\delta\Psi=\frac{i}{L}([\lambda,\Psi]+[\Lambda,\psi]+[\Lambda,\Psi]_{\text{off-diag}})\,, \qquad (3.17)$$

where λ and Λ are the diagonal and off-diagonal parts of the gauge function, respectively. At this stage, we impose boundary conditions with respect to θ on the diagonal matrix elements in eq. (3.5). Actually we choose such boundary conditions for the N string bits[9]

[9]In the large-N limit, N string bits having $p^+=1/R$ do not correspond to the wrapped supermembrane

having $p^+ = 1/R$ as

$$a_a(\theta+2\pi) = a_a(\theta)\,, \quad y_a(\theta+2\pi) = y_a(\theta)\,, \quad x_a^k(\theta+2\pi) = x_a^k(\theta)\,, \quad \psi_a(\theta+2\pi) = \psi_a(\theta)\,.$$

Then, it is natural that the off-diagonal matrix elements in eq. (3.6) also satisfy the following boundary conditions,

$$\begin{aligned} A_{ab}(\theta+2\pi) &= A_{ab}(\theta)\,, & Y_{ab}(\theta+2\pi) &= Y_{ab}(\theta)\,, \\ X_{ab}^k(\theta+2\pi) &= X_{ab}^k(\theta)\,, & \Psi_{ab}(\theta+2\pi) &= \Psi_{ab}(\theta)\,. \end{aligned}$$

Our next task is to consider the path-integral formula of the action (3.7). The gauge conditions for the diagonal and off-diagonal parts are chosen as follows,

$$a = y\,, \tag{3.18}$$

$$\partial_\theta Y - \frac{i}{L}[y,Y] - \frac{i}{L}[x^k, X^k] + \frac{i}{L}[a,A] - \partial_\tau A = 0\,. \tag{3.19}$$

We proceed in the Landau gauge. Then, we obtain the path-integral formula of the action (3.7),

$$T = \int \mathcal{D}y\mathcal{D}x^k\mathcal{D}\psi\mathcal{D}c\mathcal{D}\bar{c}\mathcal{D}A\mathcal{D}Y\mathcal{D}X^k\mathcal{D}\Psi\mathcal{D}C\mathcal{D}\bar{C}\mathcal{D}B\ \exp\left[i(S+S_{gf}+S_{gh})\right], \tag{3.20}$$

$$S+S_{gf}+S_{gh} = LT\int d\tau \int_0^{2\pi} d\theta(\mathcal{L}_B+\mathcal{L}_F+\mathcal{L}_{gf}+\mathcal{L}_{gh})\,, \tag{3.21}$$

$$\mathcal{L}_{gf} = \mathrm{tr}\left[B\left(\partial_\theta Y - \frac{i}{L}[y,Y] - \frac{i}{L}[x^k,X^k] + \frac{i}{L}[y,A] - \partial_\tau A\right)\right], \tag{3.22}$$

$$\begin{aligned} \mathcal{L}_{gh} = \mathrm{tr}\Bigg[& i\bar{c}(\partial_\tau - \partial_\theta)c - i\Bigg\{\partial_\theta\bar{C}\left(\partial_\theta C - \frac{i}{L}[y,C] - \frac{i}{L}[Y,C]\right) - \\ & - \frac{i}{L}[y,\bar{C}]\left(\partial_\theta C - \frac{i}{L}[y,C] - \frac{i}{L}[Y,C]\right) + \frac{1}{L^2}[\bar{C},Y]_{\mathrm{diag}}[C,Y]_{\mathrm{diag}} - \\ & - \frac{1}{L^2}[x^k,\bar{C}]\left([x^k,C]+[X^k,C]\right) + \frac{1}{L^2}[\bar{C},X^k]_{\mathrm{diag}}[C,X^k]_{\mathrm{diag}} + \\ & + \frac{i}{L}[y,\bar{C}]\left(\partial_\tau C - \frac{i}{L}[y,C] - \frac{i}{L}[A,C]\right) - \frac{1}{L^2}[\bar{C},A]_{\mathrm{diag}}[C,A]_{\mathrm{diag}} - \\ & - \partial_\tau\bar{C}\left(\partial_\tau C - \frac{i}{L}[y,C] - \frac{i}{L}[A,C]\right)\Bigg\}\Bigg], \end{aligned} \tag{3.23}$$

where the integration over a is carried out by using the Landau gauge condition for eq. (3.18). Note that in eq. (3.23), the coupling terms between the diagonal part of the ghost (anti-ghost) and the off-diagonal part of the anti-ghost (ghost), such as $\bar{C}c$ ($\bar{c}C$), do not exist. This is due to the Landau gauge for the gauge condition eq. (3.19). Now the off-diagonal parts are rescaled as [5]

$$A \to LA\,, \quad Y \to LY\,, \quad X^k \to LX^k\,, \quad \Psi \to L^{1/2}\Psi\,, \quad \bar{C} \to \bar{C}\,, \quad C \to L^2 C\,, \quad B \to B\,. \tag{3.24}$$

---

directly. In fact, in ref. [5], the correspondence of a long string having $p^+ = N/R$ with the wrapped supermembrane was discussed. In this paper, however, we study only the reduction from matrix strings to the N string bits for simplicity.

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Then the action (3.21) is given by

$$
\begin{aligned}
S+S_{gf}+S_{gh} &= LT\int d\tau\int_0^{2\pi}d\theta(\mathcal{L}^{\rm string}+\mathcal{L}_0^B+L\mathcal{L}_1^B+L^2\mathcal{L}_2^B+\mathcal{L}_0^F+L^{1/2}\mathcal{L}_{1/2}^F+L\mathcal{L}_1^F)\,,\\
\mathcal{L}^{\rm string} &= {\rm tr}\Big[\frac{1}{2}(\partial_\tau x^k)^2-\frac{1}{2}(\partial_\theta x^k)^2+\frac{1}{2}\left\{(\partial_\tau-\partial_\theta)y\right\}^2+\\
&\quad +i\bar{c}(\partial_\tau-\partial_\theta)c+i\psi^T\partial_\tau\psi-i\psi^T\gamma^9\partial_\theta\psi\Big]\,, \qquad (3.25)\\
\mathcal{L}_0^B &= {\rm tr}\Big[-\frac{1}{2}[x^k,A]^2+\frac{1}{2}[x^k,Y]^2+\frac{1}{2}[x^k,X^l]^2-\\
&\quad -\frac{1}{2}\Big([y,Y]+[x^k,X^k]-[y,A]\Big)^2-\\
&\quad -iB\Big([y,Y]+[x^k,X^k]-[y,A]\Big)+i[x^k,\bar{C}][x^k,C]\Big]\,, \qquad (3.26)\\
\mathcal{L}_1^B &= {\rm tr}\Big[-i\partial_\tau Y[y,Y]+2i\partial_\tau Y[y,A]-i\partial_\tau A[y,Y]+2i\partial_\theta A[y,Y]-\\
&\quad -i\partial_\theta A[y,A]-i\partial_\theta Y[y,A]-i\partial_\tau X^k[y,X^k]+2i\partial_\tau X^k[x^k,A]-\\
&\quad -i\partial_\tau A[x^k,X^k]+i\partial_\theta X^k[y,X^k]-2i\partial_\theta X^k[x^k,Y]+i\partial_\theta Y[x^k,X^k]-\\
&\quad -[y,Y][A,Y]+[y,A][A,Y]-[y,X^k][A,X^k]+[x^k,A][A,X^k]+\\
&\quad +[y,X^k][Y,X^k]-[x^k,Y][Y,X^k]+[x^k,X^l][X^k,X^l]+B\partial_\theta Y-\\
&\quad -B\partial_\tau A-\partial_\theta\bar{C}[y,C]-[y,\bar{C}]\partial_\theta C+i[y,\bar{C}][Y,C]+i[x^k,\bar{C}][X^k,C]+\\
&\quad +[y,\bar{C}]\,\partial_\tau C-i[y,\bar{C}][A,C]+\partial_\tau\bar{C}[y,C]\Big]\,, \qquad (3.27)\\
\mathcal{L}_2^B &= {\rm tr}\Big[\frac{1}{2}(\partial_\tau Y-\partial_\theta A)^2+\frac{1}{2}(\partial_\tau X^k)^2-\frac{1}{2}(\partial_\theta X^k)^2-\\
&\quad -i\partial_\tau Y[A,Y]+i\partial_\theta A[A,Y]-i\partial_\tau X^k[A,X^k]+i\partial_\theta X^k[Y,X^k]-\\
&\quad -\frac{1}{2}[A,Y]^2-\frac{1}{2}[A,X^k]^2+\frac{1}{2}[Y,X^k]^2+\frac{1}{4}[X^k,X^l]^2-i\partial_\theta\bar{C}\partial_\theta C-\\
&\quad -\partial_\theta\bar{C}[Y,C]+i\partial_\tau\bar{C}\partial_\tau C+\partial_\tau\bar{C}[A,C]+i[\bar{C},A]_{\rm diag}[C,A]_{\rm diag}-\\
&\quad -i[\bar{C},Y]_{\rm diag}[C,Y]_{\rm diag}-i[\bar{C},X^k]_{\rm diag}[C,X^k]_{\rm diag}\Big]\,, \qquad (3.28)\\
\mathcal{L}_0^F &= {\rm tr}\Big[\Psi^T[y,\Psi]-\Psi^T\gamma^9[y,\Psi]-\Psi^T\gamma^k[x^k,\Psi]\Big], \qquad (3.29)\\
\mathcal{L}_{1/2}^F &= {\rm tr}\Big[-2\Psi^T[\psi,A]+2\Psi^T\gamma^9[\psi,Y]+2\Psi^T\gamma^k[\psi,X^k]\Big], \qquad (3.30)\\
\mathcal{L}_1^F &= {\rm tr}\Big[i\Psi^T\partial_\tau\Psi-i\Psi^T\gamma^9\partial_\theta\Psi+\Psi^T[A,\Psi]-\Psi^T\gamma^9[Y,\Psi]-\Psi^T\gamma^k[X^k,\Psi]\Big]. \qquad (3.31)
\end{aligned}
$$

Our purpose is to study the behavior of matrix string theory for small radius L. Actually, by using the above action, we perform the perturbative expansion with respect to L and integrate only the off-diagonal matrix elements. The expansion is essentially the strong coupling expansion with respect to the gauge coupling $g\sim 1/L$. In the expansion, we regard eqs. (3.26) and (3.29) as the free parts and eqs. (3.27), (3.28), (3.30) and (3.31) as the interactions. In the next subsection, we read off the propagators from the free parts (3.26) and (3.29). In subsection 3.3, based on the expansion, we integrate the off-diagonal matrix elements in eq. (3.20) and derive the effective action for the diagonal matrix elements,

$$T = \int \mathcal{D}y\mathcal{D}x^k\mathcal{D}\psi\mathcal{D}c\mathcal{D}\bar{c}\exp\left(iS_{\text{eff}}[y,x^k,\bar{c},c,\psi]\right), \tag{3.32}$$

$$S_{\text{eff}}[y,x^k,\bar{c},c,\psi] = LT\int d\tau\int_0^{2\pi} d\theta\left(\mathcal{L}^{\text{string}} - i\ln Z[y,x^k,\psi]\right), \tag{3.33}$$

$$Z[y,x^k,\psi] = \int \mathcal{D}A\mathcal{D}Y\mathcal{D}X^k\mathcal{D}\Psi\mathcal{D}C\mathcal{D}\bar{C}\mathcal{D}B\exp\left[i\tilde{S}\right], \tag{3.34}$$

$$\tilde{S} = LT\int d\tau\int_0^{2\pi} d\theta\left(\mathcal{L}_0^B + L\mathcal{L}_1^B + L^2\mathcal{L}_2^B + \mathcal{L}_0^F + L^{1/2}\mathcal{L}_{1/2}^F + L\mathcal{L}_1^F\right). \tag{3.35}$$

Henceforth we set $\xi = (\tau,\theta)$ and $LT = 1$ for brevity.

3.2 Free action and propagators

From eqs. (3.26) and (3.29), the free action is given by

$$\begin{aligned}
\tilde{S}_0 &= \int d^2\xi\,(\mathcal{L}_0^B + \mathcal{L}_0^F) \\
&= \sum_{a,b=1}^{N}\int d^2\xi\Big[\frac{1}{2}(x_a^k - x_b^k)^2 A_{ab}A_{ba} - \frac{1}{2}(x_a^k - x_b^k)^2 Y_{ab}Y_{ba} - \frac{1}{2}(x_a^k - x_b^k)^2 X_{ab}^l X_{ba}^l - \\
&\qquad - \frac{1}{2}\Big\{(y_a - y_b)Y_{ab} + (x_a^k - x_b^k)X_{ab}^k - (y_a - y_b)A_{ab}\Big\}\times \\
&\qquad \times\Big\{(y_b - y_a)Y_{ba} + (x_b^k - x_a^k)X_{ba}^k - (y_b - y_a)A_{ba}\Big\} - \\
&\qquad - iB_{ab}\Big\{(y_b - y_a)Y_{ba} + (x_b^k - x_a^k)X_{ba}^k - (y_b - y_a)A_{ba}\Big\} - \\
&\qquad - i(x_a^k - x_b^k)^2\bar{C}_{ab}C_{ba} - (y_a - y_b)\Psi_{ab}^T(1-\gamma^9)\Psi_{ba} + (x_a^k - x_b^k)\Psi_{ab}^T\gamma^k\Psi_{ba}\Big],
\end{aligned} \tag{3.36}$$

where use has been made of the matrix elements in eqs. (3.5) and (3.6). Similarly we can rewrite the interaction parts (3.27), (3.28), (3.30) and (3.31) (See appendix A for the concrete expressions). From the above expression of the free action, it is easy to read off the propagators,

$$\langle Y_{ab}(\xi)Y_{ba}(\xi')\rangle = \frac{-i}{(x_a - x_b)^2}\left(1 - \frac{(y_a - y_b)^2}{(x_a - x_b)^2}\right)G(\xi,\xi'), \tag{3.37}$$

$$\langle X_{ab}^k(\xi)X_{ba}^l(\xi')\rangle = \frac{-i}{(x_a - x_b)^2}\left(\delta^{kl} - \frac{(x_a^k - x_b^k)(x_a^l - x_b^l)}{(x_a - x_b)^2}\right)G(\xi,\xi'), \tag{3.38}$$

$$\langle A_{ab}(\xi)A_{ba}(\xi')\rangle = \frac{i}{(x_a - x_b)^2}\left(1 + \frac{(y_a - y_b)^2}{(x_a - x_b)^2}\right)G(\xi,\xi'), \tag{3.39}$$

$$\langle X_{ab}^k(\xi)Y_{ba}(\xi')\rangle = \langle X_{ab}^k(\xi)A_{ba}(\xi')\rangle = i\frac{(x_a^k - x_b^k)(y_a - y_b)}{[(x_a - x_b)^2]^2}G(\xi,\xi'), \tag{3.40}$$

$$\langle A_{ab}(\xi)Y_{ba}(\xi')\rangle = i\frac{(y_a - y_b)^2}{[(x_a - x_b)^2]^2}G(\xi,\xi'), \tag{3.41}$$

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$$\langle B_{ab}(\xi)Y_{ba}(\xi')\rangle = \langle B_{ab}(\xi)A_{ba}(\xi')\rangle = \frac{y_a - y_b}{(x_a - x_b)^2}\,G(\xi,\xi')\,, \tag{3.42}$$

$$\langle B_{ab}(\xi)X^k_{ba}(\xi')\rangle = \frac{x^k_a - x^k_b}{(x_a - x_b)^2}\,G(\xi,\xi')\,, \tag{3.43}$$

$$\langle \bar{C}_{ab}(\xi)C_{ba}(\xi')\rangle = \frac{1}{(x_a - x_b)^2}\,G(\xi,\xi')\,, \tag{3.44}$$

$$\langle \Psi^\alpha_{ab}(\xi)\Psi^\beta_{ba}(\xi')\rangle = -\frac{i}{2}\left\{(y_a - y_b)(I+\gamma^9)_{\alpha\beta} + (x^k_a - x^k_b)\gamma^k_{\alpha\beta}\right\}\frac{1}{(x_a - x_b)^2}\,G(\xi,\xi')\,, \tag{3.45}$$

where $G(\xi,\xi') \equiv \delta^{(2)}(\xi-\xi') = \delta(\tau-\tau')\delta(\theta-\theta')$, $(x_a-x_b)^2 \equiv (x^k_a-x^k_b)(x^k_a-x^k_b) = (x^k_a-x^k_b)^2$ and the spinor indices α, β run through $1, 2, \ldots, 16$. Here we should notice that $(x_a - x_b)^2$ for any pairs $a \neq b$ $(1 \leq a, b \leq N)$ must be non-zero in order that the perturbative expansion with respect to L makes sense since the propagators are singular at $(x_a-x_b)^2 = 0$. We recall that in matrix string theory, the usual string interactions are described by the exchanges of coincident diagonal matrix elements, which correspond to the world-sheet instanton effects [15]. Hence, at the points where usual string interactions occur, the perturbative expansion with respect to L does not make sense even for small radius L. In this paper we consider only the situations in which the perturbative expansion with respect to L makes sense, and by using the expansion we integrate out the off-diagonal matrix elements and derive the effective action for the diagonal matrix elements. Under these circumstances where the usual string interactions are neglected, it is expected that the quantum corrections cancel out and the effective action agrees with the classical (free) action of DLCQ type-IIA superstring.

For later convenience, we define new hatted variables $\hat{X}^K$ and $\hat{x}^K$ ($K = k, 9, 10$ ($k = 1, 2, \ldots, 8$)),

$$\hat{X}^k = X^k\,, \qquad \hat{X}^9 = Y\,, \qquad \hat{X}^{10} = iA\,, \qquad \hat{x}^k = x^k\,, \qquad \hat{x}^9 = y\,, \qquad \hat{x}^{10} = iy\,. \tag{3.46}$$

The propagators (3.37)–(3.41) are represented in a single form with the new variables,

$$\langle \hat{X}^K_{ab}(\xi)\hat{X}^L_{ba}(\xi')\rangle = \frac{-i}{(\hat{x}_a - \hat{x}_b)^2}\left(\delta^{KL} - \frac{(\hat{x}^K_a - \hat{x}^K_b)(\hat{x}^L_a - \hat{x}^L_b)}{(\hat{x}_a - \hat{x}_b)^2}\right)G(\xi,\xi')\,. \tag{3.47}$$

Here note that $(\hat{x}_a - \hat{x}_b)^2 \equiv (\hat{x}^K_a - \hat{x}^K_b)(\hat{x}^K_a - \hat{x}^K_b) = (x^k_a - x^k_b)(x^k_a - x^k_b) = (x_a - x_b)^2$.

3.3 Effective action

In this subsection we calculate the effective action (3.33) (or (3.34)) based on the perturbative expansion with respect to the radius L. As is emphasized in ref. [5], however, the calculations are not well-defined. The reason is as follows: In the previous subsection we have seen that the propagators (3.37)–(3.45) in the perturbative expansion are proportional to the δ-function $G(\xi,\xi') = \delta^{(2)}(\xi - \xi')$. Hence, the loop contributions have the ultraviolet divergences like $\delta^{(2)}(0)$ and we need a regularization for the divergences. However, it is very difficult to find a suitable regularization which respects symmetries (e.g., supersymmetry and gauge symmetry) and hence we cannot perform well-defined calculations. Actually, we can easily understand a difficulty in finding the regularization. If we adopt a certain regularization (e.g., cutoff regularization for large momenta), the regularized δ-function

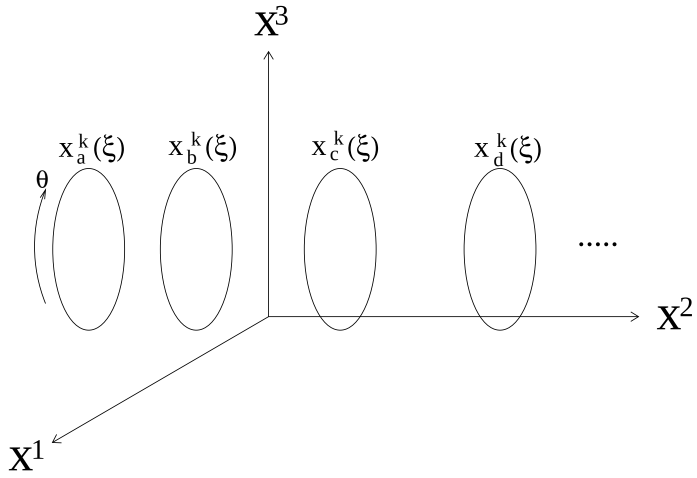


Figure 1: A configuration of N string bits in which the differences of arbitrary two bits $(x_a^k - x_b^k)$ are constant, though $x_a^k(\xi)$ and $x_b^k(\xi)$ depend on ξ in general.

$G(\xi, \xi')$ would not satisfy such a property of δ-function that $f(\xi)G(\xi, \xi') = f(\xi')G(\xi, \xi')$. Then we shall have an ambiguity how we choose the arguments in the differences of the diagonal matrix elements $(x_a^k - x_b^k)$ and $(y_a - y_b)$ which appear in the propagators (3.37)–(3.45). For example, we can choose $(x_a^k(\xi) - x_b^k(\xi))$ and $(y_a(\xi) - y_b(\xi))$, $(x_a^k(\xi') - x_b^k(\xi'))$ and $(y_a(\xi') - y_b(\xi'))$, or etc. To avoid the ambiguity, henceforth we consider only the configurations of the diagonal matrix elements in which the differences of arbitrary two elements $(x_a^k - x_b^k)$, $(y_a - y_b)$ and $(\psi_a - \psi_b)$ are independent of ξ, although x_a^k, x_b^k, y_a, y_b, ψ_a and ψ_b themselves depend on ξ in general. (See figure 1 for such a configuration.) Here we should notice that we have not yet found the suitable regularization, though we have reduced an ambiguity by restricting configurations of the diagonal matrix elements. Hence we still cannot give the well-defined calculations perfectly but only give a formal argument about the quantum corrections by studying whether the coefficients of $\delta^{(2)}(0)$'s cancel out between the bosonic and fermionic degrees of freedoms.

3.3.1 $O(L^0)$

The lowest order contribution in eq. (3.34) is the one-loop determinant of the free action (3.36). Actually, the determinant is unity due to the coincidence between bosonic and fermionic degrees of freedoms.

3.3.2 $O(L^{1/2})$

The next contribution in eq. (3.34) comes from the interaction part (A.3) of $O(L^{1/2})$. However the contribution $\langle i\tilde{S}^F_{1/2} \rangle$ vanishes because there is no way to self-contract $i\tilde{S}^F_{1/2}$ as we can see from eq. (A.3).

3.3.3 $O(L^1)$

The $O(L^1)$ contributions in eq. (3.34) come from eqs. (A.1), (A.3) and (A.4). Actually, there are tree kinds of contributions, $\langle i\tilde{S}^B_1 \rangle$, $(1/2!)\langle i\tilde{S}^F_{1/2}\, i\tilde{S}^F_{1/2} \rangle$ and $\langle i\tilde{S}^F_1 \rangle$.

First we consider $\langle i\tilde{S}_1^B\rangle$. In $\langle i\tilde{S}_1^B\rangle$, the second summation in eq. (A.1) does not contribute because there is no way to self-contract that part. Hence, we get

$$\begin{aligned}\langle i\tilde{S}_1^B\rangle = iL\int d^2\xi \sum_{a,b=1}^{N}\Big\langle & i(y_a-y_b)\partial_\tau Y_{ab}Y_{ba} - i(y_a-y_b)\partial_\tau Y_{ab}A_{ba} - i(y_a-y_b)\partial_\theta A_{ab}Y_{ba} + \\ & + i(y_a-y_b)\partial_\theta A_{ab}A_{ba} + i(y_a-y_b)\partial_\tau X^k_{ab}X^k_{ba} - \\ & - i(x^k_a-x^k_b)\partial_\tau X^k_{ab}A_{ba} - i(y_a-y_b)\partial_\theta X^k_{ab}X^k_{ba} + \\ & + i(x^k_a-x^k_b)\partial_\theta X^k_{ab}Y_{ba} + B_{ab}\partial_\theta Y_{ba} - B_{ab}\partial_\tau A_{ba} + \\ & + 2(y_a-y_b)\partial_\theta\bar{C}_{ab}C_{ba} - 2(y_a-y_b)\partial_\tau\bar{C}_{ab}C_{ba}\Big\rangle, \end{aligned} \tag{3.48}$$

where we have performed partial integrations due to the assumption that $(x^k_a - x^k_b)$ and $(y_a - y_b)$ are independent of ξ. Furthermore, we can rewrite the equation as

$$\begin{aligned}\langle i\tilde{S}_1^B\rangle = iL\int d^2\xi\int d^2\xi' \sum_{a,b=1}^{N}\Big\{ & i(y_a-y_b)\partial_\tau\langle Y_{ab}(\xi)Y_{ba}(\xi')\rangle - i(y_a-y_b)\partial_\tau\langle Y_{ab}(\xi)A_{ba}(\xi')\rangle - \\ & - i(y_a-y_b)\partial_\theta\langle A_{ab}(\xi)Y_{ba}(\xi')\rangle + i(y_a-y_b)\partial_\theta\langle A_{ab}(\xi)A_{ba}(\xi')\rangle + \\ & + i(y_a-y_b)\partial_\tau\langle X^k_{ab}(\xi)X^k_{ba}(\xi')\rangle - i(x^k_a-x^k_b)\partial_\tau\langle X^k_{ab}(\xi)A_{ba}(\xi')\rangle - \\ & - i(y_a-y_b)\partial_\theta\langle X^k_{ab}(\xi)X^k_{ba}(\xi')\rangle + i(x^k_a-x^k_b)\partial_\theta\langle X^k_{ab}(\xi)Y_{ba}(\xi')\rangle - \\ & - \partial_\theta\langle B_{ab}(\xi)Y_{ba}(\xi')\rangle + \partial_\tau\langle B_{ab}(\xi)A_{ba}(\xi')\rangle + \\ & + 2(y_a-y_b)\,\partial_\theta\langle\bar{C}_{ab}(\xi)C_{ba}(\xi')\rangle - \\ & - 2(y_a-y_b)\,\partial_\tau\langle\bar{C}_{ab}(\xi)C_{ba}(\xi')\rangle\Big\}\,\delta^{(2)}(\xi-\xi'). \end{aligned} \tag{3.49}$$

In this contribution, from the expression of the propagators (3.37)–(3.45), we see that the quantity in the braces { } is antisymmetric with respect to the exchange of the indices a and b. Hence, $\langle i\tilde{S}_1^B\rangle$ is zero by summing over a and b.

Next we consider $(1/2!)\langle i\tilde{S}^F_{1/2}\, i\tilde{S}^F_{1/2}\rangle$. From the interaction (A.3), we get

$$\begin{aligned}\frac{1}{2!}\langle i\tilde{S}^F_{1/2}\, i\tilde{S}^F_{1/2}\rangle = & -2L\int d^2\xi\int d^2\xi' \times \\ & \times\sum_{a,b=1}^{N}\Big[(\psi^\alpha_a(\xi)-\psi^\alpha_b(\xi))(\psi^{\alpha'}_a(\xi')-\psi^{\alpha'}_b(\xi'))\times \\ & \quad\times\Big\{\langle\Psi^\alpha_{ab}(\xi)\Psi^{\alpha'}_{ba}(\xi')\rangle\langle A_{ba}(\xi)A_{ab}(\xi')\rangle - \\ & \quad - \langle\Psi^\alpha_{ab}(\xi)\Psi^{\beta'}_{ba}(\xi')\rangle\langle A_{ba}(\xi)Y_{ab}(\xi')\rangle\,\gamma^9_{\beta'\alpha'} - \\ & \quad - \langle\Psi^\alpha_{ab}(\xi)\Psi^{\beta'}_{ba}(\xi')\rangle\langle A_{ba}(\xi)X^k_{ab}(\xi')\rangle\,\gamma^k_{\beta'\alpha'} - \\ & \quad - \langle\Psi^\beta_{ab}(\xi)\Psi^{\alpha'}_{ba}(\xi')\rangle\langle Y_{ba}(\xi)A_{ab}(\xi')\rangle\,\gamma^9_{\beta\alpha} + \\ & \quad + \langle\Psi^\beta_{ab}(\xi)\Psi^{\beta'}_{ba}(\xi')\rangle\langle Y_{ba}(\xi)Y_{ab}(\xi')\rangle\,\gamma^9_{\beta\alpha}\gamma^9_{\beta'\alpha'} + \\ & \quad + \langle\Psi^\beta_{ab}(\xi)\Psi^{\beta'}_{ba}(\xi')\rangle\langle Y_{ba}(\xi)X^{k'}_{ab}(\xi')\rangle\,\gamma^9_{\beta\alpha}\gamma^{k'}_{\beta'\alpha'} - \\ & \quad - \langle\Psi^\beta_{ab}(\xi)\Psi^{\alpha'}_{ba}(\xi')\rangle\langle X^k_{ba}(\xi)A_{ab}(\xi')\rangle\,\gamma^k_{\beta\alpha} + \\ & \quad + \langle\Psi^\beta_{ab}(\xi)\Psi^{\beta'}_{ba}(\xi')\rangle\langle X^k_{ba}(\xi)Y_{ab}(\xi')\rangle\,\gamma^k_{\beta\alpha}\gamma^9_{\beta'\alpha'} + \\ & \quad + \langle\Psi^\beta_{ab}(\xi)\Psi^{\beta'}_{ba}(\xi')\rangle\langle X^k_{ba}(\xi)X^{k'}_{ab}(\xi')\rangle\,\gamma^k_{\beta\alpha}\gamma^{k'}_{\beta'\alpha'}\Big\}\Big]. \end{aligned} \tag{3.50}$$

Also in this contribution, from the expression of the propagators (3.37)–(3.45), we see that the quantity in the bracket [ ] is antisymmetric with respect to the exchange of the indices a and b. Hence, it is obvious that $(1/2!)\langle i\tilde{S}^F_{1/2}\, i\tilde{S}^F_{1/2}\rangle$ is zero.

Finally, we consider $\langle i\tilde{S}^F_1\rangle$. In $\langle i\tilde{S}^F_1\rangle$, the second summation in eq. (A.4) does not contribute because there is no way to self-contract that part. Then we get

$$\begin{aligned}\langle i\tilde{S}^F_1\rangle &= iL\int d^2\xi \sum_{a,b=1}^{N}\left\{ i\langle \Psi^T_{ab}\partial_\tau \Psi_{ba}\rangle - i\langle \Psi^T_{ab}\gamma^9\partial_\theta \Psi_{ba}\rangle\right\} \\ &= iL\int d^2\xi \int d^2\xi' \sum_{a,b=1}^{N}\Big\{ i\partial_\tau\langle \Psi^\alpha_{ab}(\xi)\Psi^\alpha_{ba}(\xi')\rangle - \\ &\qquad\qquad - i\partial_\theta\langle \Psi^\alpha_{ab}(\xi)\Psi^{\alpha'}_{ba}(\xi')\rangle\,\gamma^9_{\alpha\alpha'}\Big\}\delta^2(\xi-\xi')\,. \end{aligned} \tag{3.51}$$

Also in this contribution, from the expression of the propagators (3.37)–(3.45), we see that the quantity in the braces { } is antisymmetric with respect to the exchange of the indices a and b. Hence, it is obvious that $\langle i\tilde{S}^F_1\rangle$ is zero. Thus, all quantum corrections of $O(L)$ to the classical string action are zero. Note that to show the zero quantum corrections of $O(L)$, we have used only the antisymmetry under the exchange of the indices a and b, and we have never used the fact that $G(\xi,\xi')$, which appears in the propagators, is the δ-function. Hence the quantum corrections of $O(L)$ would be zero even if we adopt a certain regularization and $G(\xi,\xi')$ is the regularized δ-function.

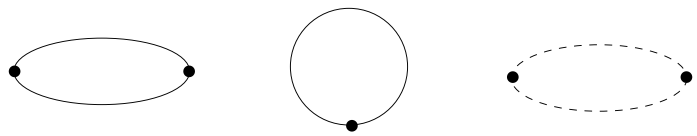
3.3.4 $O(L^{3/2})$

At $O(L^{3/2})$, there are tree kinds of contributions in eq. (3.34). Those are $(1/2!)\langle i\tilde{S}^B_1\, i\tilde{S}^F_{1/2}\rangle$, $(1/3!)\langle i\tilde{S}^F_{1/2}\, i\tilde{S}^F_{1/2}\, i\tilde{S}^F_{1/2}\rangle$ and $(1/2!)\langle i\tilde{S}^F_1\, i\tilde{S}^F_{1/2}\rangle$. However, each contribution is zero because there is no way of contraction in $i\tilde{S}^B_1\, i\tilde{S}^F_{1/2}$, $i\tilde{S}^F_{1/2}\, i\tilde{S}^F_{1/2}\, i\tilde{S}^F_{1/2}$ and $i\tilde{S}^F_1\, i\tilde{S}^F_{1/2}$, respectively.

3.3.5 $O(L^2)$

At $O(L^2)$, there are many contributions in eq. (3.34). Actually, those are $(1/2!)\langle i\tilde{S}^B_1 i\tilde{S}^B_1\rangle$, $\langle i\tilde{S}^B_2\rangle$, $(1/2!)\langle i\tilde{S}^F_1\, i\tilde{S}^F_1\rangle$, $(1/2!)\langle i\tilde{S}^B_1 i\tilde{S}^F_1\rangle$, $(1/3!)\langle i\tilde{S}^F_{1/2} i\tilde{S}^F_{1/2} i\tilde{S}^B_1\rangle$, $(1/3!)\langle i\tilde{S}^F_{1/2} i\tilde{S}^F_{1/2} i\tilde{S}^F_1\rangle$ and $(1/4!)\langle i\tilde{S}^F_{1/2} i\tilde{S}^F_{1/2} i\tilde{S}^F_{1/2} i\tilde{S}^F_{1/2}\rangle$. Note that the last three contributions contain fermionic diagonal elements ψ_a. They each vanish due to the anti-commutativity of the Grassmann variable ψ_a. And it is easy to show that the contribution of $(1/2!)\langle i\tilde{S}^B_1 i\tilde{S}^F_1\rangle$ vanishes. Then $(1/2!)\langle i\tilde{S}^B_1 i\tilde{S}^B_1\rangle$, $\langle i\tilde{S}^B_2\rangle$ and $(1/2!)\langle i\tilde{S}^F_1\, i\tilde{S}^F_1\rangle$ give non-trivial contributions in eq. (3.34), which will be calculated below.

(1) One-loop contributions. First, we consider the one-loop contributions in $(1/2!)\times\langle i\tilde{S}^B_1 i\tilde{S}^B_1\rangle$, $\langle i\tilde{S}^B_2\rangle$ and $(1/2!)\langle i\tilde{S}^F_1\, i\tilde{S}^F_1\rangle$, which are referred to as $(1/2!)\langle i\tilde{S}^B_1 i\tilde{S}^B_1\rangle^{(\text{one-loop})}$, $\langle i\tilde{S}^B_2\rangle^{(\text{one-loop})}$ and $(1/2!)\langle i\tilde{S}^F_1\, i\tilde{S}^F_1\rangle^{(\text{one-loop})}$, respectively. In figure 2, we give Feynman diagrams which correspond to such contributions.



(a) (b) (c)

Figure 2: One-loop Feynman diagrams at $O(L^2)$. Figures (a), (b) and (c) correspond to one-loop contributions in $(1/2!)\langle i\tilde{S}_1^B i\tilde{S}_1^B\rangle$, $\langle i\tilde{S}_2^B\rangle$ and $(1/2!)\langle i\tilde{S}_1^F i\tilde{S}_1^F\rangle$, respectively. Solid lines denote the propagators of the bosonic or the ghost fields and dashed lines denote those of the fermionic fields.

First of all we consider $(1/2!)\langle i\tilde{S}_1^B i\tilde{S}_1^B\rangle^{(\text{one-loop})}$, which is calculated as

$$
\begin{aligned}
\frac{1}{2!}\langle i\tilde{S}_1^B i\tilde{S}_1^B\rangle^{(\text{one-loop})} &= -\frac{L^2}{2}\int d^2\xi \int d^2\xi' \times \\
&\times \sum_{a,b=1}^{N}\sum_{a',b'=1}^{N}\Big\langle \Big\{ i(y_a-y_b)\partial_\tau Y_{ab}Y_{ba}(\xi) - i(y_a-y_b)\partial_\tau Y_{ab}A_{ba}(\xi) - \\
&\quad - i(y_a-y_b)\partial_\theta A_{ab}Y_{ba}(\xi) + i(y_a-y_b)\partial_\theta A_{ab}A_{ba}(\xi) + \\
&\quad + i(y_a-y_b)\partial_\tau X^k_{ab}X^k_{ba}(\xi) - i(x^k_a-x^k_b)\partial_\tau X^k_{ab}A_{ba}(\xi) - \\
&\quad - i(y_a-y_b)\partial_\theta X^k_{ab}X^k_{ba}(\xi) + i(x^k_a-x^k_b)\partial_\theta X^k_{ab}Y_{ba}(\xi) + \\
&\quad + B_{ab}\partial_\theta Y_{ba}(\xi) - B_{ab}\partial_\tau A_{ba}(\xi) + \\
&\quad + 2(y_a-y_b)\partial_\theta \bar{C}_{ab}C_{ba}(\xi) - 2(y_a-y_b)\partial_\tau \bar{C}_{ab}C_{ba}(\xi)\Big\} \times \\
&\quad \times \Big\{ i(y_{a'}-y_{b'})\partial_{\tau'} Y_{a'b'}Y_{b'a'}(\xi') - i(y_{a'}-y_{b'})\partial_{\tau'} Y_{a'b'}A_{b'a'}(\xi') - \\
&\quad - i(y_{a'}-y_{b'})\partial_{\theta'} A_{a'b'}Y_{b'a'}(\xi') + i(y_{a'}-y_{b'})\partial_{\theta'} A_{a'b'}A_{b'a'}(\xi') + \\
&\quad + i(y_{a'}-y_{b'})\partial_{\tau'} X^{k'}_{a'b'}X^{k'}_{b'a'}(\xi') - i(x^{k'}_{a'}-x^{k'}_{b'})\partial_{\tau'} X^{k'}_{a'b'}A_{b'a'}(\xi') - \\
&\quad - i(y_{a'}-y_{b'})\partial_{\theta'} X^{k'}_{a'b'}X^{k'}_{b'a'}(\xi') + i(x^{k'}_{a'}-x^{k'}_{b'})\partial_{\theta'} X^{k'}_{a'b'}Y_{b'a'}(\xi') + \\
&\quad + B_{a'b'}\partial_{\theta'} Y_{b'a'}(\xi') - B_{a'b'}\partial_{\tau'} A_{b'a'}(\xi') + \\
&\quad + 2(y_{a'}-y_{b'})\partial_{\theta'} \bar{C}_{a'b'}C_{b'a'}(\xi') - 2(y_{a'}-y_{b'})\partial_{\tau'} \bar{C}_{a'b'}C_{b'a'}(\xi')\Big\}\Big\rangle \\
&= L^2\int d^2\xi\int d^2\xi' \sum_{a\neq b}\Bigg[\left(\frac{1}{(x_a-x_b)^2}+\frac{17}{2}\frac{(y_a-y_b)^2}{\{(x_a-x_b)^2\}^2}\right)\partial_\tau\partial_{\tau'}G(\xi,\xi')\,G(\xi,\xi') - \\
&\quad - 17\,\frac{(y_a-y_b)^2}{\{(x_a-x_b)^2\}^2}\partial_\tau\partial_{\theta'}G(\xi,\xi')\,G(\xi,\xi') - \\
&\quad - \left(\frac{1}{(x_a-x_b)^2}-\frac{17}{2}\frac{(y_a-y_b)^2}{\{(x_a-x_b)^2\}^2}\right)\partial_\theta\partial_{\theta'}G(\xi,\xi')\,G(\xi,\xi')\Bigg].
\end{aligned}
\tag{3.52}
$$

Next we consider $\langle i\tilde{S}_2^B\rangle^{(\text{one-loop})}$ and it is also calculated as

$$
\begin{aligned}
&\langle i\tilde{S}_2^B\rangle^{(\text{one-loop})} = \\
&= iL^2\int d^2\xi\sum_{a,b=1}^{N}\Big\{\frac{1}{2}\langle\partial_\tau Y_{ab}\partial_\tau Y_{ba}\rangle - \langle\partial_\tau Y_{ab}\partial_\theta A_{ba}\rangle + \frac{1}{2}\langle\partial_\theta A_{ab}\partial_\theta A_{ba}\rangle + \frac{1}{2}\langle\partial_\tau X^k_{ab}\partial_\tau X^k_{ba}\rangle - \\
&\qquad - \frac{1}{2}\langle\partial_\theta X^k_{ab}\partial_\theta X^k_{ba}\rangle - i\langle\partial_\theta\bar{C}_{ab}\partial_\theta C_{ba}\rangle + i\langle\partial_\tau\bar{C}_{ab}\partial_\tau C_{ba}\rangle\Big\}
\end{aligned}
$$

$$
= L^2 \int d^2\xi \int d^2\xi' \sum_{a\neq b} \Bigg[\left(\frac{3}{(x_a - x_b)^2} - \frac{1}{2} \frac{(y_a - y_b)^2}{\{(x_a - x_b)^2\}^2} \right) \partial_\tau \partial_{\tau'} G(\xi, \xi')\, \delta^{(2)}(\xi - \xi') + \\
+ \frac{(y_a - y_b)^2}{\{(x_a - x_b)^2\}^2} \partial_\tau \partial_{\theta'} G(\xi, \xi')\, \delta^{(2)}(\xi - \xi') - \\
- \left(\frac{3}{(x_a - x_b)^2} + \frac{1}{2} \frac{(y_a - y_b)^2}{\{(x_a - x_b)^2\}^2} \right) \partial_\theta \partial_{\theta'} G(\xi, \xi')\, \delta^{(2)}(\xi - \xi') \Bigg] . \tag{3.53}
$$

Finally, we consider $(1/2!)\langle i\tilde{S}_1^F i\tilde{S}_1^F \rangle^{\text{(one-loop)}}$. It is given by

$$
\frac{1}{2!} \langle i\tilde{S}_1^F i\tilde{S}_1^F \rangle^{\text{(one-loop)}} =
$$

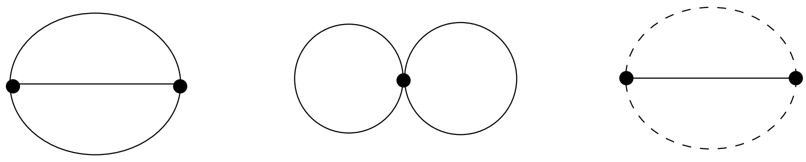
$$
= -\frac{L^2}{2} \int d^2\xi \int d^2\xi' \sum_{a,b=1}^{N} \sum_{a',b'=1}^{N} \Big\langle \{ i\Psi_{ab}^T \partial_\tau \Psi_{ba}(\xi) - i\Psi_{ab}^T \gamma^9 \partial_\theta \Psi_{ba}(\xi) \} \times \\
\times \{ i\Psi_{a'b'}^T \partial_{\tau'} \Psi_{b'a'}(\xi') - i\Psi_{a'b'}^T \gamma^9 \partial_{\theta'} \Psi_{b'a'}(\xi') \} \Big\rangle
$$

$$
= L^2 \int d^2\xi \int d^2\xi' \sum_{a\neq b} \Bigg[\left(\frac{-4}{(x_a - x_b)^2} - 8 \frac{(y_a - y_b)^2}{\{(x_a - x_b)^2\}^2} \right) \partial_\tau \partial_{\tau'} G(\xi, \xi')\, G(\xi, \xi') + \\
+ 16 \frac{(y_a - y_b)^2}{\{(x_a - x_b)^2\}^2} \partial_\tau \partial_{\theta'} G(\xi, \xi')\, G(\xi, \xi') + \\
+ \left(\frac{4}{(x_a - x_b)^2} - 8 \frac{(y_a - y_b)^2}{\{(x_a - x_b)^2\}^2} \right) \partial_\theta \partial_{\theta'} G(\xi, \xi')\, G(\xi, \xi') \Bigg] . \tag{3.54}
$$

Note that we have never used the fact that $G(\xi, \xi')$ is the δ-function in calculating eqs. (3.52)–(3.54). Hence eqs. (3.52)–(3.54) is expected to be unaltered even if we adopt a certain regularization and $G(\xi, \xi')$ is a regularized δ-function. We first use the fact that $G(\xi, \xi')$ is the δ-function at this stage and it is shown that the one-loop quantum correction at $O(L^2)$ is zero, i.e., $(1/2!)\langle i\tilde{S}_1^B i\tilde{S}_1^B \rangle^{\text{(one-loop)}} + \langle i\tilde{S}_2^B \rangle^{\text{(one-loop)}} + (1/2!)\langle i\tilde{S}_1^F\, i\tilde{S}_1^F \rangle^{\text{(one-loop)}} = 0$. Of course the above calculations in matrix string theory are essentially the same as the ones in the wrapped supermembrane theory [5].[10]

(2) Two-loop contributions. Next, we consider the two-loop contributions in $(1/2!) \times \langle i\tilde{S}_1^B i\tilde{S}_1^B \rangle$, $\langle i\tilde{S}_2^B \rangle$ and $(1/2!)\langle i\tilde{S}_1^F\, i\tilde{S}_1^F \rangle$, which are not calculated in ref. [5]. We refer to them as $(1/2!)\langle i\tilde{S}_1^B i\tilde{S}_1^B \rangle^{\text{(two-loop)}}$, $\langle i\tilde{S}_2^B \rangle^{\text{(two-loop)}}$ and $(1/2!)\langle i\tilde{S}_1^F\, i\tilde{S}_1^F \rangle^{\text{(two-loop)}}$, respectively. In figure 3, we give Feynman diagrams corresponding to them.

[10] In ref. [5], the zero-mode gauge field a is restricted to be zero by hand in calculating the one-loop quantum corrections. In this paper, however, we have just fixed the gauge ($a = y$) and added the corresponding FP-ghost part following the standard procedure [16]. In this sense the configuration of the gauge field a is not restricted in our calculations.



(a) (b) (c)

Figure 3: Two-loop Feynman diagrams at $O(L^2)$. Figures (a), (b) and (c) correspond to two-loop contributions in $(1/2!)\langle i\tilde{S}_1^B i\tilde{S}_1^B\rangle$, $\langle i\tilde{S}_2^B\rangle$ and $(1/2!)\langle i\tilde{S}_1^F\, i\tilde{S}_1^F\rangle$, respectively.

First, we consider $(1/2!)\langle i\tilde{S}_1^B i\tilde{S}_1^B\rangle^{(\text{two-loop})}$, which is given by

$$\begin{aligned}
\frac{1}{2!}\langle i\tilde{S}_1^B i\tilde{S}_1^B\rangle^{(\text{two-loop})} &= -\frac{L^2}{2}\int d^2\xi \int d^2\xi' \times \\
\times \sum_{a,b,c=1}^{N}\sum_{a',b',c'=1}^{N} \Bigg\langle \Big\{ &-(y_a-y_b)Y_{ab}(A_{bc}Y_{ca}-Y_{bc}A_{ca})(\xi)+ \\
&+(y_a-y_b)A_{ab}(A_{bc}Y_{ca}-Y_{bc}A_{ca})(\xi)- \\
&-(y_a-y_b)X^k_{ab}(A_{bc}X^k_{ca}-X^k_{bc}A_{ca})(\xi)+ \\
&+(x^k_a-x^k_b)A_{ab}(A_{bc}X^k_{ca}-X^k_{bc}A_{ca})(\xi)+ \\
&+(y_a-y_b)X^k_{ab}(Y_{bc}X^k_{ca}-X^k_{bc}Y_{ca})(\xi)- \\
&-(x^k_a-x^k_b)Y_{ab}(Y_{bc}X^k_{ca}-X^k_{bc}Y_{ca})(\xi)+ \\
&+(x^k_a-x^k_b)X^l_{ab}(X^k_{bc}X^l_{ca}-X^l_{bc}X^k_{ca})(\xi)- \\
&-i(y_a-y_b)\bar{C}_{ab}(A_{bc}C_{ca}-C_{bc}A_{ca})(\xi)+ \\
&+i(y_a-y_b)\bar{C}_{ab}(Y_{bc}C_{ca}-C_{bc}Y_{ca})(\xi)+ \\
&+i(x^k_a-x^k_b)\bar{C}_{ab}(X^k_{bc}C_{ca}-C_{bc}X^k_{ca})(\xi)\Big\}\times \\
\times\Big\{ &-(y_{a'}-y_{b'})\,Y_{a'b'}(A_{b'c'}Y_{c'a'}-Y_{b'c'}A_{c'a'})(\xi')+ \\
&+(y_{a'}-y_{b'})\,A_{a'b'}(A_{b'c'}Y_{c'a'}-Y_{b'c'}A_{c'a'})(\xi')- \\
&-(y_{a'}-y_{b'})\,X^{k'}_{a'b'}(A_{b'c'}X^{k'}_{c'a'}-X^{k'}_{b'c'}A_{c'a'})(\xi')+ \\
&+(x^{k'}_{a'}-x^{k'}_{b'})\,A_{a'b'}(A_{b'c'}X^{k'}_{c'a'}-X^{k'}_{b'c'}A_{c'a'})(\xi')+ \\
&+(y_{a'}-y_{b'})\,X^{k'}_{a'b'}(Y_{b'c'}X^{k'}_{c'a'}-X^{k'}_{b'c'}Y_{c'a'})(\xi')- \\
&-(x^{k'}_{a'}-x^{k'}_{b'})\,Y_{a'b'}(Y_{b'c'}X^{k'}_{c'a'}-X^{k'}_{b'c'}Y_{c'a'})(\xi')+ \\
&+(x^{k'}_{a'}-x^{k'}_{b'})\,X^{l'}_{a'b'}(X^{k'}_{b'c'}X^{l'}_{c'a'}-X^{l'}_{b'c'}X^{k'}_{c'a'})(\xi')- \\
&-i(y_{a'}-y_{b'})\,\bar{C}_{a'b'}(A_{b'c'}C_{c'a'}-C_{b'c'}A_{c'a'})(\xi')+ \\
&+i(y_{a'}-y_{b'})\,\bar{C}_{a'b'}(Y_{b'c'}C_{c'a'}-C_{b'c'}Y_{c'a'})(\xi')+ \\
&+i(x^{k'}_{a'}-x^{k'}_{b'})\,\bar{C}_{a'b'}(X^{k'}_{b'c'}C_{c'a'}-C_{b'c'}X^{k'}_{c'a'})(\xi')\Big\}\Bigg\rangle . \qquad (3.55)
\end{aligned}$$

By using the variables $\hat{X}^K$ and $\hat{x}^K$ in eq. (3.46) and the propagator (3.47), we can put the above expression into a compact form and we obtain

$$\frac{1}{2!}\langle i\tilde{S}_1^B i\tilde{S}_1^B\rangle^{(\text{two-loop})} =$$

$$\begin{aligned}
&= -\frac{L^2}{2}\int d^2\xi\int d^2\xi' \sum_{a,b,c=1}^{N}\sum_{a',b',c'=1}^{N}\Big\langle\Big\{(\hat{x}_a^K-\hat{x}_b^K)\hat{X}_{ab}^L(\hat{X}_{bc}^K\hat{X}_{ca}^L-\hat{X}_{bc}^L\hat{X}_{ca}^K)(\xi)+\\
&\qquad + i(\hat{x}_a^K-\hat{x}_b^K)\bar{C}_{ab}(\hat{X}_{bc}^K C_{ca}-C_{bc}\hat{X}_{ca}^K)(\xi)\Big\}\times\\
&\qquad \times\Big\{(\hat{x}_{a'}^{K'}-\hat{x}_{b'}^{K'})\hat{X}_{a'b'}^{L'}(\hat{X}_{b'c'}^{K'}\hat{X}_{c'a'}^{L'}-\hat{X}_{b'c'}^{L'}\hat{X}_{c'a'}^{K'})(\xi')\\
&\qquad + i(\hat{x}_{a'}^{K'}-\hat{x}_{b'}^{K'})\bar{C}_{a'b'}(\hat{X}_{b'c'}^{K'}C_{c'a'}-C_{b'c'}\hat{X}_{c'a'}^{K'})(\xi')\Big\}\Big\rangle\\
&= -iL^2\int d^2\xi\int d^2\xi' \sum_{a\neq b,\, b\neq c,\, c\neq a}\Bigg\{\frac{33}{2}\frac{1}{(\hat{x}_a-\hat{x}_b)^2(\hat{x}_b-\hat{x}_c)^2}-\\
&\qquad -16\,\frac{\{(\hat{x}_a^K-\hat{x}_b^K)(\hat{x}_c^K-\hat{x}_a^K)\}^2}{(\hat{x}_a-\hat{x}_b)^2(\hat{x}_b-\hat{x}_c)^2\{(\hat{x}_c-\hat{x}_a)^2\}^2}-\\
&\qquad -\frac{1}{2}\,\frac{\{(\hat{x}_a^K-\hat{x}_b^K)(\hat{x}_b^K-\hat{x}_c^K)\}^2}{\{(\hat{x}_a-\hat{x}_b)^2(\hat{x}_b-\hat{x}_c)^2\}^2}\Bigg\}(G(\xi,\xi'))^3\\
&= -iL^2\int d^2\xi\int d^2\xi' \sum_{a\neq b,\, b\neq c,\, c\neq a}\Bigg\{\frac{33}{2}\frac{1}{(x_a-x_b)^2(x_b-x_c)^2}-\\
&\qquad -16\,\frac{\{(x_a^k-x_b^k)(x_c^k-x_a^k)\}^2}{(x_a-x_b)^2(x_b-x_c)^2\{(x_c-x_a)^2\}^2}-\\
&\qquad -\frac{1}{2}\,\frac{\{(x_a^k-x_b^k)(x_b^k-x_c^k)\}^2}{\{(x_a-x_b)^2(x_b-x_c)^2\}^2}\Bigg\}(G(\xi,\xi'))^3\,.
\end{aligned}\tag{3.56}$$

Next, we consider $\langle i\tilde{S}_2^B\rangle^{(\text{two-loop})}$,

$$\langle i\tilde{S}_2^B\rangle^{(\text{two-loop})} =$$

$$\begin{aligned}
&= iL^2\int d^2\xi\Bigg[\sum_{a,b,c,d=1}^{N}\Big\{-\langle A_{ab}Y_{bc}A_{cd}Y_{da}\rangle+\langle A_{ab}Y_{bc}Y_{cd}A_{da}\rangle-\langle A_{ab}X_{bc}^k A_{cd}X_{da}^k\rangle+\\
&\qquad +\langle A_{ab}X_{bc}^k X_{cd}^k A_{da}\rangle+\langle Y_{ab}X_{bc}^k Y_{cd}X_{da}^k\rangle-\langle Y_{ab}X_{bc}^k X_{cd}^k Y_{da}\rangle+\\
&\qquad +\frac{1}{2}\langle X_{ab}^k X_{bc}^l X_{cd}^k X_{da}^l\rangle-\frac{1}{2}\langle X_{ab}^k X_{bc}^l X_{cd}^l X_{da}^k\rangle\Big\}+\\
&\quad +\sum_{a,b,c=1}^{N}\Big\{i\langle\bar{C}_{ab}A_{ba}C_{ac}A_{ca}\rangle-i\langle\bar{C}_{ab}A_{ba}A_{ac}C_{ca}\rangle-i\langle A_{ab}\bar{C}_{ba}C_{ac}A_{ca}\rangle+\\
&\qquad +i\langle A_{ab}\bar{C}_{ba}A_{ac}C_{ca}\rangle-i\langle\bar{C}_{ab}Y_{ba}C_{ac}Y_{ca}\rangle+i\langle\bar{C}_{ab}Y_{ba}Y_{ac}C_{ca}\rangle+\\
&\qquad +i\langle Y_{ab}\bar{C}_{ba}C_{ac}Y_{ca}\rangle-i\langle Y_{ab}\bar{C}_{ba}Y_{ac}C_{ca}\rangle-i\langle\bar{C}_{ab}X_{ba}^k C_{ac}X_{ca}^k\rangle+\\
&\qquad +i\langle\bar{C}_{ab}X_{ba}^k X_{ac}^k C_{ca}\rangle+i\langle X_{ab}^k\bar{C}_{ba}C_{ac}X_{ca}^k\rangle-i\langle X_{ab}^k\bar{C}_{ba}X_{ac}^k C_{ca}\rangle\Big\}\Bigg]\,.
\end{aligned}\tag{3.57}$$

By using the variables $\hat{X}^K$ and $\hat{x}^K$ in eq. (3.46) and the propagator (3.47), we can also put the above expression into a compact form and we get

$$
\begin{aligned}
&\langle i\tilde{S}_2^B\rangle^{(\text{two-loop})} = \\
&= iL^2 \int d^2\xi \Bigg[\sum_{a,b,c,d=1}^{N} \left\{ \frac{1}{2}\langle \hat{X}^K_{ab}\hat{X}^L_{bc}\hat{X}^K_{cd}\hat{X}^L_{da}\rangle - \frac{1}{2}\langle \hat{X}^K_{ab}\hat{X}^L_{bc}\hat{X}^L_{cd}\hat{X}^K_{da}\rangle \right\} + \\
&\qquad + \sum_{a,b,c=1}^{N} \Big\{ -i\langle \bar{C}_{ab}\hat{X}^K_{ba}C_{ac}\hat{X}^K_{ca}\rangle + i\langle \bar{C}_{ab}\hat{X}^K_{ba}\hat{X}^K_{ac}C_{ca}\rangle + \\
&\qquad\qquad + i\langle \hat{X}^K_{ab}\bar{C}_{ba}C_{ac}\hat{X}^K_{ca}\rangle - i\langle \hat{X}^K_{ab}\bar{C}_{ba}\hat{X}^K_{ac}C_{ca}\rangle \Big\} \Bigg] \\
&= iL^2 \int d^2\xi \int d^2\xi' \Bigg[\sum_{a\neq b,\, b\neq c} \left\{ \frac{73}{2}\frac{1}{(\hat{x}_a-\hat{x}_b)^2(\hat{x}_b-\hat{x}_c)^2} - \frac{1}{2}\frac{\{(\hat{x}^K_a-\hat{x}^K_b)(\hat{x}^K_b-\hat{x}^K_c)\}^2}{\{(\hat{x}_a-\hat{x}_b)^2(\hat{x}_b-\hat{x}_c)^2\}^2} \right\} + \\
&\qquad + 18\sum_{a\neq b}\frac{1}{\{(\hat{x}_a-\hat{x}_b)^2\}^2} \Bigg] (G(\xi,\xi'))^2\, \delta^{(2)}(\xi-\xi') \\
&= iL^2 \int d^2\xi \int d^2\xi' \Bigg[\sum_{a\neq b,\, b\neq c} \left\{ \frac{73}{2}\frac{1}{(x_a-x_b)^2(x_b-x_c)^2} - \frac{1}{2}\frac{\{(x^k_a-x^k_b)(x^k_b-x^k_c)\}^2}{\{(x_a-x_b)^2(x_b-x_c)^2\}^2} \right\} + \\
&\qquad + 18\sum_{a\neq b}\frac{1}{\{(x_a-x_b)^2\}^2} \Bigg] (G(\xi,\xi'))^2\, \delta^{(2)}(\xi-\xi')\,. \qquad (3.58)
\end{aligned}
$$

The first and second summations in the above equation correspond to the third and forth summations in eq. (A.2), respectively. Now we extract the $a = c$ part from the first summation in eq. (3.58) and add it to the second summation. Then we get

$$
\begin{aligned}
&\langle i\tilde{S}_2^B\rangle^{(\text{two-loop})} = \\
&= iL^2 \int d^2\xi \int d^2\xi' \Bigg[\sum_{a\neq b,\, b\neq c,\, c\neq a} \left\{ \frac{73}{2}\frac{1}{(x_a-x_b)^2(x_b-x_c)^2} - \frac{1}{2}\frac{\{(x^k_a-x^k_b)(x^k_b-x^k_c)\}^2}{\{(x_a-x_b)^2(x_b-x_c)^2\}^2} \right\} + \\
&\qquad + 54\sum_{a\neq b}\frac{1}{\{(x_a-x_b)^2\}^2} \Bigg] (G(\xi,\xi'))^2\, \delta^{(2)}(\xi-\xi')\,. \qquad (3.59)
\end{aligned}
$$

Finally we consider $(1/2!)\langle i\tilde{S}_1^F\, i\tilde{S}_1^F\rangle^{(\text{two-loop})}$ and the contribution is calculated as

$$
\begin{aligned}
&\frac{1}{2!}\langle i\tilde{S}_1^F i\tilde{S}_1^F\rangle^{(\text{two-loop})} = \\
&= -\frac{L^2}{2}\int d^2\xi \int d^2\xi' \sum_{a,b,c=1}^{N}\sum_{a',b',c'=1}^{N} \Big\langle \Big\{ \Psi^T_{ab}(A_{bc}\Psi_{ca}-\Psi_{bc}A_{ca})(\xi) - \\
&\qquad - \Psi^T_{ab}\gamma^9(Y_{bc}\Psi_{ca}-\Psi_{bc}Y_{ca})(\xi) - \\
&\qquad - \Psi^T_{ab}\gamma^k(X^k_{bc}\Psi_{ca}-\Psi_{bc}X^k_{ca})(\xi)\Big\} \times \\
&\qquad \times \Big\{ \Psi^T_{a'b'}(A_{b'c'}\Psi_{c'a'}-\Psi_{b'c'}A_{c'a'})(\xi') - \\
&\qquad - \Psi^T_{a'b'}\gamma^9(Y_{b'c'}\Psi_{c'a'}-\Psi_{b'c'}Y_{c'a'})(\xi') - \\
&\qquad - \Psi^T_{a'b'}\gamma^{k'}(X^{k'}_{b'c'}\Psi_{c'a'}-\Psi_{b'c'}X^{k'}_{c'a'})(\xi')\Big\} \Big\rangle
\end{aligned}
$$

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$$= -iL^2 \int d^2\xi \int d^2\xi' \sum_{a\neq b,\, b\neq c,\, c\neq a} \left\{ 20 \frac{1}{(x_a - x_b)^2 (x_b - x_c)^2} + \right. \\ \left. + 16 \frac{\{(x_a^k - x_b^k)(x_c^k - x_a^k)\}^2}{(x_a - x_b)^2 (x_b - x_c)^2 \{(x_c - x_a)^2\}^2} \right\} (G(\xi,\xi'))^3 . \tag{3.60}$$

Note that in calculating eqs. (3.56), (3.59) and (3.60) we have never used the fact that $G(\xi,\xi')$ is the δ-function. Hence eqs. (3.56), (3.59) and (3.60) are expected to be unaltered even if we adopt a certain regularization and $G(\xi,\xi')$ is a regularized δ-function. We first use the fact that $G(\xi,\xi')$ is the δ-function at this stage and sum up eqs. (3.56), (3.59) and (3.60). Then, we obtain

$$\frac{1}{2!}\langle i\tilde{S}_1^B i\tilde{S}_1^B\rangle^{(\text{two-loop})} + \langle i\tilde{S}_2^B\rangle^{(\text{two-loop})} + \frac{1}{2!}\langle i\tilde{S}_1^F \, i\tilde{S}_1^F\rangle^{(\text{two-loop})} = \\ = iL^2 \int d^2\xi \int d^2\xi' \sum_{a\neq b} \frac{54}{\{(x_a - x_b)^2\}^2} (G(\xi,\xi'))^3 . \tag{3.61}$$

Thus we see that the two-loop quantum corrections at $O(L^2)$ do not cancel out. One comment is in order: The remaining term is exactly that of the second summation in eq. (3.59). If we assume that the differences of the diagonal elements can be estimated as $(x_a^k - x_b^k) \sim O(N^\alpha)$[11] with some common constant α when N is large, we will see that the terms canceled in eq. (3.61), i.e., terms given by the summations over a, b and c with $a \neq b$, $b \neq c$, $c \neq a$ in eqs. (3.56), (3.59) and (3.60), behave as $\sum_{a\neq b,\, b\neq c,\, c\neq a}(x_a^k - x_b^k)^{-2}(x_b^k - x_c^k)^{-2} \sim O(N^{3-4\alpha})$, while the remaining term, which comes from the second summation in eq. (3.59), behaves as $\sum_{a\neq b}(x_a^k - x_b^k)^{-4} \sim O(N^{2-4\alpha})$. In this sense, we could say that only the leading terms in the large N can be canceled out in the two-loop quantum corrections to the classical string action at $O(L^2)$.

It will be pedagogical to re-consider the results (3.56), (3.59) and (3.60) in the case of $N = 2$. In this case, it is obvious that eqs. (3.56) and (3.60) are zero. The reason is as follows: Schematically, each term in eqs. (3.56) and (3.60) is represented by $\langle \mathrm{tr}(X^3)\, \mathrm{tr}(X^3)\rangle$, where X stands for a bosonic or fermionic 2×2 matrix of only off-diagonal components, i.e., its diagonal components are zero. Thus $\mathrm{tr}(X^3) = 0$ and hence eqs. (3.56) and (3.60) are zero. On the other hand, each term in eq. (3.59) is schematically represented by $\langle \mathrm{tr}(X^4)\rangle$ and it can have non-zero value. Thus in $N = 2$ case, it is obvious that only the bosonic contribution of eq. (3.59) exists.

4. Conclusion and discussion

In this paper we have studied in matrix string theory whether the reduction to the diagonal elements of the matrices is justified quantum mechanically. We have seen that at $O(L^2)$, the two-loop quantum corrections do not cancel out. Our calculations are essentially two-loop extension of the previous ones in ref. [5].

[11] According to the correspondence of a long string in matrix string theory with the wrapped supermembrane given in ref. [5], $\alpha = -1$ for $|a - b| \ll N$.

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We should note that no suitable regularization for the divergences of $\delta^{(2)}(0)$ type is found so far, and hence we have only studied a mechanism of cancellations of the divergences between bosonic and fermionic degrees of freedoms. Actually, we have found that at the two-loop level of $O(L^2)$, the sub-leading term in the large N appears only from the bosonic degrees of freedom and cannot be canceled out. Even if we find a suitable regularization, such a structure seems to be unaltered and hence our result will be unchanged.

Finally, we comment on the global constraints (2.14) and (2.15) in the wrapped supermembrane theory. To be precise, such constraints should be taken into account in the calculations of the quantum double-dimensional reduction.[12] In matrix string theory, however, there are no counterparts of such constraints, as was discussed in section 2. In particular, in the standard derivation of matrix string theory, they do not appear naturally. However, our result may suggest that the suitably matrix-regularized constraints should be incorporated with the standard form of matrix string theory.

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A. Interaction part of the action

In this appendix we give the interaction part of the action by using the matrix elements in eqs. (3.5) and (3.6),

$$
\begin{aligned}
\tilde{S}_1^B &= L \int d^2\xi\, \mathcal{L}_1^B \\
&= L \int d^2\xi \Bigg[\sum_{a,b=1}^{N} \Big\{ i(y_a - y_b)\partial_\tau Y_{ab} Y_{ba} - 2i(y_a - y_b)\partial_\tau Y_{ab} A_{ba} + i(y_a - y_b)\partial_\tau A_{ab} Y_{ba} - \\
&\qquad - 2i(y_a - y_b)\partial_\theta A_{ab} Y_{ba} + i(y_a - y_b)\partial_\theta A_{ab} A_{ba} + i(y_a - y_b)\partial_\theta Y_{ab} A_{ba} + \\
&\qquad + i(y_a - y_b)\partial_\tau X^k_{ab} X^k_{ba} - 2i(x^k_a - x^k_b)\partial_\tau X^k_{ab} A_{ba} + i(x^k_a - x^k_b)\partial_\tau A_{ab} X^k_{ba} - \\
&\qquad - i(y_a - y_b)\partial_\theta X^k_{ab} X^k_{ba} + 2i(x^k_a - x^k_b)\partial_\theta X^k_{ab} Y_{ba} - i(x^k_a - x^k_b)\partial_\theta Y_{ab} X^k_{ba} + \\
&\qquad + B_{ab}\partial_\theta Y_{ba} - B_{ab}\partial_\tau A_{ba} + (y_a - y_b)\partial_\theta \bar{C}_{ab} C_{ba} - (y_a - y_b)\bar{C}_{ab}\partial_\theta C_{ba} + \\
&\qquad + (y_a - y_b)\bar{C}_{ab}\partial_\tau C_{ba} - (y_a - y_b)\partial_\tau \bar{C}_{ab} C_{ba} \Big\} + \\
&\quad + \sum_{a,b,c=1}^{N} \Big\{ -(y_a - y_b) Y_{ab}(A_{bc} Y_{ca} - Y_{bc} A_{ca}) + \\
&\qquad + (y_a - y_b) A_{ab}(A_{bc} Y_{ca} - Y_{bc} A_{ca}) - \\
&\qquad - (y_a - y_b) X^k_{ab}(A_{bc} X^k_{ca} - X^k_{bc} A_{ca}) + \\
&\qquad + (x^k_a - x^k_b) A_{ab}(A_{bc} X^k_{ca} - X^k_{bc} A_{ca}) + \\
&\qquad + (y_a - y_b) X^k_{ab}(Y_{bc} X^k_{ca} - X^k_{bc} Y_{ca}) - \\
&\qquad - (x^k_a - x^k_b) Y_{ab}(Y_{bc} X^k_{ca} - X^k_{bc} Y_{ca}) +
\end{aligned}
$$

[12] In ref. [5], the global constraints are not considered in their calculations.

$$
\begin{aligned}
&+ (x_a^k - x_b^k) X_{ab}^l (X_{bc}^k X_{ca}^l - X_{bc}^l X_{ca}^k) - \\
&- i(y_a - y_b)\bar{C}_{ab}(A_{bc}C_{ca} - C_{bc}A_{ca}) + \\
&+ i(y_a - y_b)\bar{C}_{ab}(Y_{bc}C_{ca} - C_{bc}Y_{ca}) + \\
&\left. \left. + i(x_a^k - x_b^k)\bar{C}_{ab}(X_{bc}^k C_{ca} - C_{bc}X_{ca}^k) \right\} \right] ,
\end{aligned} \tag{A.1}
$$

$$
\begin{aligned}
\tilde{S}_2^B &= L^2 \int d^2\xi \, \mathcal{L}_2^B \\
&= L^2 \int d^2\xi \left[\sum_{a,b=1}^{N} \left\{ \frac{1}{2}\partial_\tau Y_{ab}\partial_\tau Y_{ba} - \partial_\tau Y_{ab}\partial_\theta A_{ba} + \frac{1}{2}\partial_\theta A_{ab}\partial_\theta A_{ba} + \frac{1}{2}\partial_\tau X_{ab}^k \partial_\tau X_{ba}^k - \right. \right. \\
&\qquad \left. - \frac{1}{2}\partial_\theta X_{ab}^k \partial_\theta X_{ba}^k - i\partial_\theta \bar{C}_{ab}\partial_\theta C_{ba} + i\partial_\tau \bar{C}_{ab}\partial_\tau C_{ba} \right\} + \\
&\quad + \sum_{a,b,c=1}^{N} \left\{ i\partial_\theta A_{ab}(A_{bc}Y_{ca} - Y_{bc}A_{ca}) - i\partial_\tau Y_{ab}(A_{bc}Y_{ca} - Y_{bc}A_{ca}) - \right. \\
&\qquad - i\partial_\tau X_{ab}^k (A_{bc}X_{ca}^k - X_{bc}^k A_{ca}) + i\partial_\theta X_{ab}^k (Y_{bc}X_{ca}^k - X_{bc}^k Y_{ca}) - \\
&\qquad \left. - \partial_\theta \bar{C}_{ab}(Y_{bc}C_{ca} - C_{bc}Y_{ca}) + \partial_\tau \bar{C}_{ab}(A_{bc}C_{ca} - C_{bc}A_{ca}) \right\} + \\
&\quad + \sum_{a,b,c,d=1}^{N} \left\{ -A_{ab}Y_{bc}A_{cd}Y_{da} + A_{ab}Y_{bc}Y_{cd}A_{da} - A_{ab}X_{bc}^k A_{cd}X_{da}^k + \right. \\
&\qquad + A_{ab}X_{bc}^k X_{cd}^k A_{da} + Y_{ab}X_{bc}^k Y_{cd}X_{da}^k - Y_{ab}X_{bc}^k X_{cd}^k Y_{da} + \\
&\qquad \left. + \frac{1}{2}X_{ab}^k X_{bc}^l X_{cd}^k X_{da}^l - \frac{1}{2}X_{ab}^k X_{bc}^l X_{cd}^l X_{da}^k \right\} + \\
&\quad + \sum_{a,b,c=1}^{N} \left\{ i\bar{C}_{ab}A_{ba}C_{ac}A_{ca} - i\bar{C}_{ab}A_{ba}A_{ac}C_{ca} - iA_{ab}\bar{C}_{ba}C_{ac}A_{ca} + \right. \\
&\qquad + iA_{ab}\bar{C}_{ba}A_{ac}C_{ca} - i\bar{C}_{ab}Y_{ba}C_{ac}Y_{ca} + i\bar{C}_{ab}Y_{ba}Y_{ac}C_{ca} + \\
&\qquad + iY_{ab}\bar{C}_{ba}C_{ac}Y_{ca} - iY_{ab}\bar{C}_{ba}Y_{ac}C_{ca} - i\bar{C}_{ab}X_{ba}^k C_{ac}X_{ca}^k + \\
&\qquad \left. \left. + i\bar{C}_{ab}X_{ba}^k X_{ac}^k C_{ca} + iX_{ab}^k \bar{C}_{ba}C_{ac}X_{ca}^k - iX_{ab}^k \bar{C}_{ba}X_{ac}^k C_{ca} \right\} \right] ,
\end{aligned} \tag{A.2}
$$

$$
\begin{aligned}
\tilde{S}_{1/2}^F &= L^{1/2} \int d^2\xi \, \mathcal{L}_{1/2}^F \\
&= L^{1/2} \int d^2\xi \left[\sum_{a,b=1}^{N} \left\{ 2\Psi_{ab}^T(\psi_a - \psi_b)A_{ba} - 2\Psi_{ab}^T\gamma^9(\psi_a - \psi_b)Y_{ba} - \right. \right. \\
&\qquad \left. \left. - 2\Psi_{ab}^T\gamma^k(\psi_a - \psi_b)X_{ba}^k \right\} \right] ,
\end{aligned} \tag{A.3}
$$

$$
\begin{aligned}
\tilde{S}_1^F &= L \int d^2\xi \, \mathcal{L}_1^F \\
&= L \int d^2\xi \left[\sum_{a,b=1}^{N} \left\{ i\Psi_{ab}^T\partial_\tau\Psi_{ba} - i\Psi_{ab}^T\gamma^9\partial_\theta\Psi_{ba} \right\} + \right. \\
&\quad + \sum_{a,b,c=1}^{N} \left\{ \Psi_{ab}^T(A_{bc}\Psi_{ca} - \Psi_{bc}A_{ca}) - \Psi_{ab}^T\gamma^9(Y_{bc}\Psi_{ca} - \Psi_{bc}Y_{ca}) - \right. \\
&\qquad \left. \left. - \Psi_{ab}^T\gamma^k(X_{bc}^k\Psi_{ca} - \Psi_{bc}X_{ca}^k) \right\} \right] .
\end{aligned} \tag{A.4}
$$

The following formulas are useful to obtain the above expressions,

$$([x, X])_{ab} = (x_a - x_b)\, X_{ab}\,, \tag{A.5}$$

$$\mathrm{tr}(x[X, Y]) = \mathrm{tr}(X[Y, x]) = \mathrm{tr}(Y[x, X]) = \sum_{a,b=1}^{N} (x_b - x_a) Y_{ab} X_{ba}\,. \tag{A.6}$$

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